

Scattering of an elastic wave by a rigid sphere in a semi-bounded domain

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The problem of scattering of plane elastic waves by a rigid sphere near a rigid boundary is considered. This leads to the appearance of multiply re-reflected dilatation and shear waves, which generate strong oscillations of the wave field. The problem for a vector operator of the shear waves is reduced to the definition of scalar functions as a consequence of symmetry. Approximate formulas for the far field and the long-wave Rayleigh approximation are presented. The construction of multiply re-reflected waves by the image method is presented and analyzed. Calculations of the scattered wave fields are plotted in the form of scattering diagrams.

Keywords: wave scattering, rigid sphere, semi-bounded region, image method, far field, the Rayleigh approximation

Introduction. The problem of scattering of plane elastic waves by a rigid sphere near a rigid boundary is considered. This leads to the appearance of multiply re-reflected dilatation and shear waves, which generate strong oscillations of the wave field. The problem for a vector operator of the shear waves is reduced to the definition of scalar functions as a consequence of symmetry. Approximate formulas for the far field and the long-wave Rayleigh approximation are presented. The construction of multiply re-reflected waves by the image method is presented and analyzed. Calculations of the scattered wave fields are plotted in the form of scattering diagrams

1. Well-known works on the wave diffraction by obstacles are given in (Selezov et al., 2018)

[1] and cites only it is necessary for the problem under consideration. Foreign particles are in most materials and it can lead to strong oscillations of the wave field due to the appearance of re-reflected waves. Rigid inclusions can be in human biotissues as well. In these cases, a complex wave field of re-reflected waves arises leading to oscillations. This problem is modeled here as a problem of scattering waves by a rigid spherical inclusion near a flat rigid boundary.

The problems of diffraction of waves by inhomogeneities were considered in the case of radial inhomogeneities in (Selezov, 1993) [2]. Application of addition theorems

in multiply connected domains is presented in (Selezov et al., 2018) [2]. In (Jackson, 1962) [3] a useful presentation of the image method is given. The construction of multiple scattered fields is presented in (Selezov et al., 2018) [2]. The basic relations in the spherical coordinate system are considered in (Kratzer & Franz, 1963) [4], for cylindrical functions in (Watson, 1945) [5]. The addition theorem for spherical functions is given in (Friedman & Russek, 1954) [6], for cylindrical functions in (Watson, 1945) [5]. Diffraction of plane waves by a rigid sphere in infinite domain was considered in (Knopoff, 1959) [7], where previous studies in the field of seismology are noted, and in particular in (Ying & Truell, 1956) [8]. Diffraction of elastic waves by an elastic sphere was considered in (Jain & Kanwal, 1980)[9]. This article presents the statement of the problem of diffraction of elastic waves by a rigid spherical inclusion located near a flat rigid boundary. The transformation of a vector field to the definition of scalar functions is considered. The far-fields and approximate solutions in the long-wave Rayleigh approximation are constructed. It is shown how to construct a primary and secondary fields from geometric considerations using the image method. Calculations of the re-reflected wave fields are carried out and their strong oscillations are shown.

2. Statement of the problem

We consider a spherical coordinate system (radial, zenith and azimuthal coordinates) corresponding to a rectangular Cartesian coordinate system (Fig. 1). The axis Oy is perpendicular to the flat boundary with the origin of coordinates in the center of an rigid spherical inclusion (scatterer) and is directed from infinity to the flat boundary. When plane waves propagate from infinity (plane waves propagate along the Oy axis), a diffracted field of multiply re-reflected waves occurs in the system.

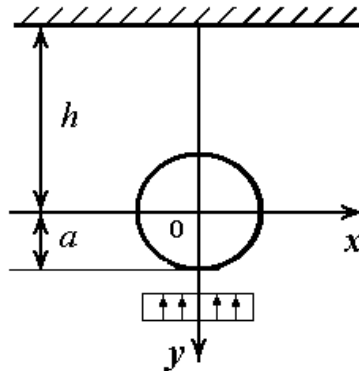


Fig. 1. Geometry of the problem

From infinity, in the direction of the axis Oy, a plane wave displacement propagates

$$u_y(0, y, 0, t) = u_0 e^{i(py + \omega t)}. \quad (1)$$

The motion of an elastic medium is described by the equations

$$\left(\nabla^2 - \frac{1}{c_e^2} \frac{\partial^2}{\partial t^2} \right) \psi = 0, \quad \left(\nabla^2 - \frac{1}{c_s^2} \frac{\partial^2}{\partial t^2} \right) \vec{a} = 0, \quad (2)$$

and the displacement vector is defined by the formula

$$\vec{u} = \vec{\nabla} \psi + \vec{\nabla} \times \vec{a}, \quad \vec{\nabla} \cdot \vec{a} = 0. \quad (3)$$

The boundary conditions on the sphere and on the flat boundary are

$$u_r|_{r=a} = 0, \quad u_\theta|_{r=a} = 0, \quad u_y|_{y=-h} = 0, \quad \sigma_{xy}|_{y=-h} = 0. \quad (4)$$

Conditions (4) mean that the displacement vector on the surface of the sphere $r = a$ is zero, and on the surface of the flat boundary $y = -h$ only a normal component of the displacement takes place while the shear stress are zero (slip). The required functions must also satisfy the conditions of Sommerfeld radiation. Dimensionless quantities are realized everywhere. The characteristic values are: length [m] is the radius of a sphere a , time [s] is $1/\omega$, kilogram-mass [kg] is Young's modulus E . Thus, the dimensionless distance from the center of the sphere to the flat boundary is h/a . In the far field approximation, the quantity h/a is assumed large $\frac{h}{a} \gg 1$, and in the Rayleigh long-wave approximation, the quantities pa and qa are small $pa \ll 1, qa \ll 1$.

The expression for the incident wave (1) corresponds to the dilatation waves, so from (1) and (3) we get

$$\psi = \frac{1}{ip} u_0 e^{i(py + \omega t)} + f(x, 0, z, t) + \text{const}. \quad (5)$$

The potential ψ is determined to an arbitrary function, which can be taken $f \equiv 0$ and written in accordance with (2) the equation for the incident wave

$$\left(\frac{\partial^2}{\partial y^2} - \frac{1}{c_e^2} \frac{\partial^2}{\partial t^2} \right) \psi(y, t) = 0. \quad (6)$$

Hence, the function ψ in (6) defines the potential of the incident wave.

Equations (2) follow from the equations of elastodynamics $G\nabla^2\vec{u} + (\lambda + G)\vec{\nabla}(\vec{\nabla}\cdot\vec{u}) = \rho\frac{\partial^2\vec{u}}{\partial t^2}$

using a known formula $\vec{\nabla}\times\vec{\nabla}\times\vec{a} = \vec{\nabla}(\vec{\nabla}\cdot\vec{a}) - \nabla^2\vec{a}$. As a result, the definition of the operator $\vec{\nabla}\times\vec{a}$ B (3) is reduced to $\nabla^2\vec{a}$ (Morse & Feshbach) [10]

$$\begin{aligned} \nabla^2\vec{a} = & \vec{e}_r \left[\nabla^2 a_r - \frac{2}{r^2} a_r - \frac{2}{r^2 \sin\theta} \frac{\partial}{\partial\theta} (\sin\theta a_\theta) - \frac{2}{r^2 \sin\theta} \frac{\partial a_\varphi}{\partial\varphi} \right] + \\ & + \vec{e}_\theta \left[\nabla^2 a_\theta - \frac{a_\theta}{r^2 \sin^2\theta} + \frac{2}{r^2} \frac{\partial a_r}{\partial\theta} - \frac{2\cos\theta}{r^2 \sin^2\theta} \frac{\partial a_\varphi}{\partial\varphi} \right] + \\ & + \vec{e}_\varphi \left[\nabla^2 a_\varphi - \frac{a_\varphi}{r^2 \sin^2\theta} + \frac{2}{r^2 \sin\theta} \frac{\partial a_r}{\partial\varphi} + \frac{2\cos\theta}{r^2 \sin^2\theta} \frac{\partial a_\theta}{\partial\varphi} \right], \end{aligned} \quad (7)$$

In the case of axial symmetry $\frac{\partial(\bullet)}{\partial\varphi} = 0$ we obtain from (7)

$$\begin{aligned} \nabla^2\vec{a} = & \vec{e}_r \left[\nabla^2 a_r - \frac{2}{r^2} a_r - \frac{2}{r^2 \sin\theta} \frac{\partial}{\partial\theta} (\sin\theta a_\theta) \right] + \\ & + \vec{e}_\theta \left[\nabla^2 a_\theta - \frac{a_\theta}{r^2 \sin^2\theta} + \frac{2}{r^2} \frac{\partial a_r}{\partial\theta} \right] + \vec{e}_\varphi \left[\nabla^2 a_\varphi - \frac{a_\varphi}{r^2 \sin^2\theta} \right]. \end{aligned} \quad (8)$$

The third term in expression (8) is also zero, since $a_\varphi = 0$. The components a_φ are the projections of the vector \vec{a} to the coordinate line φ and are zero in the case of axial symmetry. This can be established from physical considerations: shear deformations are possible only in planes normal to lines φ .

For a dilatation field (expansion - compression waves), the problem is solved simply

(Seismic diffraction, 2016) [11]. In the case of the equivoluminal field characterizing the tangential stresses, the problem is more complicated. When a flat incident wave is incident on a spherical scatterer in the direction of the Oy axis, the elastic displacements due to axial symmetry, i.e. shear stresses and corresponding deformations will be only in directions orthogonal to the coordinate line \vec{e}_φ . As a result, we obtain that the construction of an equation for ψ , one can introduce a scalar function $\xi(r, \theta)$ with normalization u_0 depending on two scalar arguments r and θ , i.e. we obtain the scalar wave equation for the expression $\nabla^2\vec{a}$ in (8) also depends on

two coordinates r and θ . By analogy with the scalar function $\xi(r, \theta)$

$$\left(\nabla^2 - \frac{1}{c_s^2} \frac{\partial^2}{\partial t^2} \right) \xi = 0$$

from which, after the separation of variables, the Legendre equation and the Bessel equation for spherical functions follow. To implement the image method in the semi-bounded region using the above formulas, the well-known solution of the diffraction of elastic waves on a sphere in the infinite region is applied (Knopoff, 1959) [7].

3. Approximate solution

Solutions in the infinite domain for functions ψ and ξ are written as

$$\psi = \sum_{m=0}^{\infty} \left[f_m j_m(pr) + a_m h_m^{(2)}(pr) \right] P_m(\cos\theta), \quad \xi = \sum_{m=0}^{\infty} b_m h_m^{(2)}(qr) \frac{\partial}{\partial \theta} P_m(\cos\theta), \quad (9)$$

where $f_m = -(2m+1) u_0 p^{-1} i^{-(m+1)}$, $j_m(pr)$ and $h_m^{(2)}(pr)$ are the spherical functions of Bessel and Hankel. For example, $j_m(\zeta) = J_{m+\frac{1}{2}}(\zeta) \sqrt{\frac{\pi\zeta}{2}}$, $j_m(\zeta) = J_{m+\frac{1}{2}}(\zeta) \sqrt{\frac{\pi\zeta}{2}}$.

The first two boundary conditions (4) are written as $\left[\frac{\partial \psi}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta a_\varphi) \right]_{r=a} = 0$, $\left[\frac{1}{r} \frac{\partial \psi}{\partial \theta} + \frac{1}{r} \left(-\frac{\partial}{\partial r} r a_\varphi \right) \right]_{r=a} = 0$ whence taking into account (9) we get the coefficients a_m and b_m

$$\begin{aligned} a_m &= f_m \Delta_m^{-1} \{ m(m+1) j_m(pa) h_m(qa) - pa j'_m(pa) [h_m(qa) + qa h'_m(qa)] \} \\ b_m &= f_m \Delta_m^{-1} pa \{ h_m(pa) j'_m(pa) - h'_m(pa) j_m(pa) \} \end{aligned} \quad (10)$$

$$\Delta_m = pa h'_m(pa) [h_m(qa) + qa h'_m(qa)] - m(m+1) h_m(pa) h_m(qa)$$

Approximate solutions for the field in the far field $\frac{r}{a} \gg 1$ are found by representing the Hankel functions by their asymptotic expansions for large r/a ,

$$u_r \cong \sum_{m=0}^{\infty} a_m i^m \frac{1}{r} e^{-ipr} P_m(\cos \theta), \quad u_{\theta} \cong - \sum_{m=0}^{\infty} b_m i^m \frac{1}{r} e^{-iqr} \frac{\partial}{\partial \theta} P_m(\cos \theta). \quad (11)$$

The pa values correspond to the ratios of the size of the scatterer a to the wavelength l . The smaller the pa , the better the series converge, and for large pa the solution in the series is inapplicable and it is necessary to pass to high-frequency asymptotics. In the work (Knopoff, 1959) [7], scatterplots of waves with different values of pa are given, which leads in some cases to incorrect results that do not satisfy the Rayleigh approximation $pa \ll 1$, and the series do not converge well.

In the Rayleigh approximation, the inequalities $pa \ll 1$ $qa \ll 1$ are valid. In this case, from (10) and (11) it can be established that the coefficients a_1 and b_1 are dominant

$$a_1 \cong i \ 3a \left[1 + 2 \left(\frac{q}{p} \right)^2 \right]^{-1}, \quad b_1 \cong i \ 3a \left[1 + 2 \left(\frac{q}{p} \right)^2 \right]^{-1} \left(\frac{q}{p} \right)^2 \quad (12)$$

Relations (12) are valid at small values $\frac{a}{h} \ll 1$ in accordance with the Rayleigh approximation.

The solution satisfying the two second boundary conditions (4) in each k th approximation is represented in the form

$$\bar{u}(r, \theta, r^*, \theta^*) = \sum_{k=1}^{\infty} \left[\bar{u}_k(r, \theta) + \bar{u}_k^*(r^*, \theta^*) \right], \quad (13)$$

where the total displacement components for the scattered multiplicity field of the multiplicity k are

$$\begin{aligned} (\bar{u}_k + \bar{u}_k^*)_r &= U_{rk} = u_{rk} - u_{rk}^* \cos(\theta + \theta^*) + u_{\theta k}^* \sin(\theta + \theta^*), \\ U_{\theta k} &= u_{\theta k} + u_{\theta k}^* \cos(\theta + \theta^*) - u_{rk}^* \sin(\theta + \theta^*). \end{aligned} \quad (14)$$

The difference in distances from the real and imaginary obstacles to a certain point r, θ and the difference in time between the arrival of P - and S - waves in the first approximation are taken into account by the formulas

$$\exp(-i\alpha_j) = (\cos \eta_j - i \sin \eta_j) \exp(-ipr), \quad \alpha_0 = qr, \quad \alpha_1 = pr^*, \quad \alpha_2 = qr^*,$$

$$\eta_0 = pr \left(\frac{q}{P} - 1 \right), \quad \eta_1 = \eta_p = pr(\eta - 1), \quad \eta_2 = \eta_q = pr \left(\frac{q}{p} \eta - 1 \right). \quad (15)$$

Formulas (15) follow from the geometrical relations obtained below for main and mirror obstacles. According to formulas (12) — (15), after a series of transformations for a single scattered field,, we find

$$U_{r1} \cong -\cos \theta + \left(\frac{2h}{r} \cos \theta - 1 \right) \left(\frac{2h}{r} - \cos \theta \right) (\cos \eta_p - i \sin \eta_p) \eta^{-3} + \left(\frac{q}{p} \right)^2 (\cos \eta_q - i \sin \eta_q) \frac{2h}{r} \eta^{-3} \sin^2 \theta, \quad (16)$$

$$U_{\theta 1} \cong \left[- \left(\frac{q}{p} \right)^2 (\cos \eta_q - i \sin \eta_q) - \left(\frac{2h}{r} \cos \theta - 1 \right) (\cos \eta_q - i \sin \eta_q) \eta^{-3} \left(\frac{q}{p} \right)^2 - \left(\frac{2h}{r} - \cos \theta \right) (\cos \eta_p - i \sin \eta_p) \frac{2h}{r} \eta^{-3} \right] \sin \theta. \quad (17)$$

In the right-hand sides of (16), (17), the multiplier $\frac{a}{r} \exp(ipr)$ is omitted, the left-hand sides are normalized by a multiplier $3 \left[1 + 2(q/p)^2 \right]^{-1}$, and displacements are referred to u_0 . The field of the incident wave and the corresponding field of the wave reflected from the boundary have the form $u_y = u_y^i - u_y^* = \exp(ipy) - \exp[i(py - 2ph)]$ have the form:. The value η is determined from formulas (18), (19) presented below.

Thus, a single diffracted field is described by formulas (16), (17). The first approximation includes the field from the incident wave (before reflection from the flat boundary in the coordinate system with origin in $z = 0$) and the field from the reflected waves from the flat boundary to scattering (this field is described in the coordinate system with origin in $z^* = 0$), which includes only normal components in connection with slippage (the fourth condition in (2)). Waves scattered on a sphere, reach the border, these scattered waves are reflected from the border and again scattered on the sphere. It is all the same that waves in image coordinates (r^*, θ^*, φ^*) are scattered, but they must be converted into coordinates r, θ, φ by the addition theorem. This completes the construction of solutions of the primary field.

4. Geometric relations and the second approximation
Note that the real and mirror fields are defined in different coordinate systems. In some cases, the mirror field can be expressed by means of geometric transformations and the use of addition theorems in the variables of the main field. We present the basic

formulas for an obstacle characterized by functions $Z_n(pr^*) \exp(in\theta^*)$ (see Fig. 2). In the case of axial symmetry (diffraction of plane waves), it is sufficient to consider the change in the plane with the meridional line θ , but using the addition theorems for spherical or cylindrical functions.

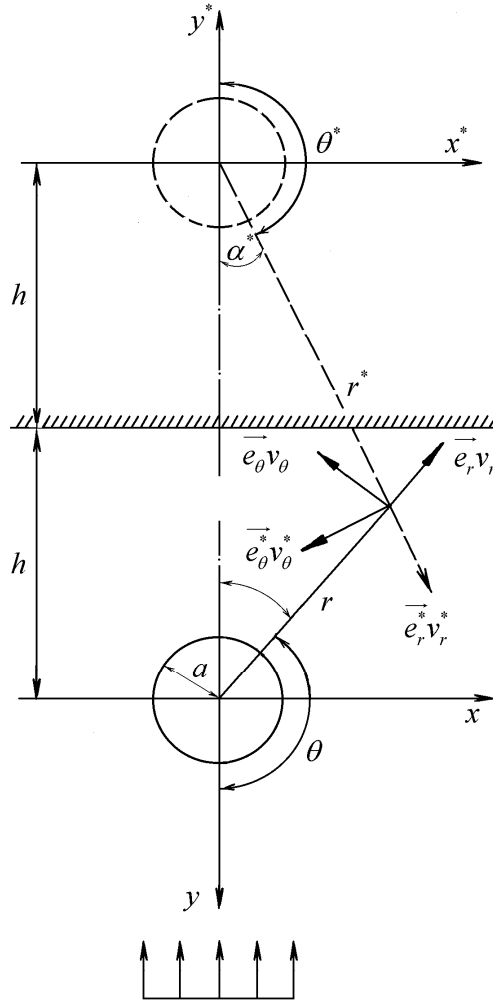


Fig. 2. On the problem of scattering of a plane wave by an obstacle in a semi-bounded region.

$$r^* = \eta r = \left[1 + 4 \left(\frac{h}{r} \right)^2 + 4 \frac{h}{r} \cos \theta \right]^{\frac{1}{2}} r$$

$$\sin n\theta^* = \sum_{k=0}^{m_1} (-1)^k C_n^{2k+1} \eta^{-n} \left(-\frac{2h}{r} - \cos\theta \right)^{n-2k-1} \sin^{2k+1} \theta,$$

$$\cos n\theta^* = \sum_{k=0}^{m_2} (-1)^k C_n^{2k} \eta^{-n} \left(-\frac{2h}{r} - \cos\theta \right)^{n-2k} \sin^{2k} \theta \quad (18)$$

$$\text{at } n = 0, 1 \quad m_1 = m_2 = 0,$$

$$\text{at } n \geq 2 \quad \frac{n-2}{2} \leq m_1 \leq \frac{n-1}{2}, \quad \frac{n-1}{2} \leq m_2 \leq \frac{n}{2}.$$

Here C_n^{2k+1} and C_n^{2k} are the binomial coefficients. From formula (18) you can establish

$$\frac{r^*}{a} = \frac{r}{a} \eta; \quad \eta = \left[1 + 4 \left(\frac{h}{r} \right)^2 + 4 \frac{h}{r} \cos\theta \right]^{\frac{1}{2}},$$

$$\sin\theta^* = \eta \sin\theta, \quad \cos\theta^* = -\eta^{-1} \left(\frac{2h}{r} + \cos\theta \right), \quad (19)$$

$$\sin(\theta + \theta^*) = -\eta^{-1} \frac{2h}{r} \sin\theta, \quad \cos(\theta + \theta^*) = -\eta^{-1} \frac{2h}{r} (\cos\theta + 1).$$

The secondary diffracted field consists of two parts. The incident field for the first part can be taken in the form of waves scattered on an imaginary obstacle and reaching the actual obstacle. In the second approximation, it is necessary to consider the scattered field on the spherical inclusion in the field of the waves reflected from the flat boundary, again the arrival of these waves to the flat boundary and the reflection from the flat boundary to the second scattering. The waves reflected from the flat boundary can be approximated by a flat wave and we get the solution of the problem again. In this case, the geometric relations will be the same for each multiplicity.

The approximate solution of the scattering problem in the second approximation is represented as

$$u_r \cong U_{r1} + U_{r2}; \quad u_\theta \cong U_{\theta1} + U_{\theta2}.$$

In the constructed approximations, the real and mirror fields are defined in different coordinate systems and in some cases, the mirror field can be expressed by means of geometric transformations and the use of addition theorems in the variables of the main field.

The addition theorem for spherical wave functions is given in (Friedman & Russek, 1954) [6]. They considered a theorem for converting the functions $j_n(kR)P_n^{|m|}(\cos\theta)e^{im\varphi}$, $h_n^{(1)}(kR)P_n^{|m|}(\cos\theta)e^{im\varphi}$, $h_n^{(2)}(kR)P_n^{|m|}(\cos\theta)e^{im\varphi}$ from point P to point O'. The same type of formulas can be obtained for other coordinate systems.

As an example, we find the displacements u_r and u_θ with the following data: Poisson's ratio $\nu = 0,25$; $pa = 0,18$; $r/a = 200$. With the selected parameters, the error of the applied formulas does not exceed 10%. The calculations are performed in points with a step $\pi/36$. The calculation results are shown in Fig. 3, from which oscillations of re-reflected waves are clearly visible.

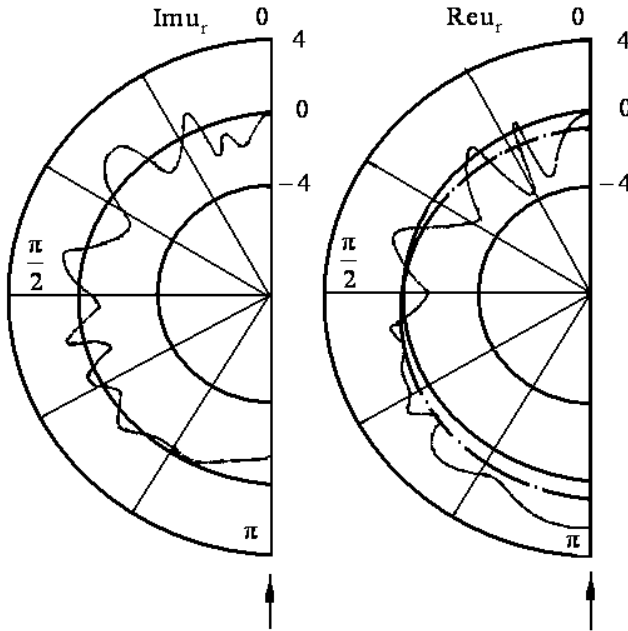


Fig. 3. The change in the values of the scattered field Imu_r and Reu_r with $h/a = 200$ (solid line) and with $h/r = \infty$ (dash-dotted line) without flat boundary. It is of interest to carry out calculations in the case of a one-term approximation for the quantities: $pa = 0.10, \frac{q}{p} = 1.870$ with $\nu = 0.3$; $pa = 0.10, \frac{q}{p} = 3.317$ at $\nu = 0.45$, and also to estimate the modules of the coefficients a_1, b_1 . In the case of a two-term approximation, the data given above can be applied.

5. Conclusion

In this article, we have investigated using the image method the problem of plane elastic wave scattering by a rigid sphere located at some distance from a flat rigid boundary. On the sphere surface the conditions of zero displacements are satisfied, and on the flat boundary, the conditions for slippage are satisfied, i.e. a zero tangent stresses. Exact solutions in spherical functions have written in an infinite region, from which approximate solutions are obtained in the case of the far field (a large distance of the sphere from the flat boundary) and the long-wave Rayleigh approximation. As a result, solutions describing a multiply re-reflected wave field for the primary and

secondary fields were obtained. Formulas from geometric relations were derived and using them the single and secondary fields were constructed. Calculations have been carried out and scattering diagrams plotted showing a strongly oscillating wave field.

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Розсіювання пружних хвиль на жорсткій сфері в напівобмеженій області

Igor Selezov

Розглянуто задачу розсіювання плоских пружних хвиль на жорсткій сфері біля жорсткої границі. Це призводить до появи багаторазово відбитих дилатаційних і поперечних хвиль, які генерують сильні коливання хвильового поля. Задача для векторного оператора поперечних хвиль зводиться до визначення скалярних функцій як наслідок симетрії. Наведено наближені формули для поля в дальній зоні і довгохвильового наближення Релея. Подано і проаналізовано побудову багаторазово перевідбитих хвиль методом зображень. Розрахунки розсіяних хвильових полів представлені у вигляді діаграм розсіювання.

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